Geometric Series

Monday, April 10, 2023 9:28 AM

geometric series: choose a real value $r \in \mathbb{R}$

geometric series is series in form r^n

- geometric series for r is $\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + r^4 + \ldots$

starts @ $n=0$

$$\left(\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ if } |r| < 1 \right)$$

$a_n = 1/2^n$ $\quad$ $a_n = 3^n$

$r' = r = 1/2$ (converged) $\quad$ $r = 3$ (diverged)

needs to go from $0 \to \infty$ to work

* - lemma: $\sum_{n=0}^{\infty} r^n = \begin{cases} \text{exist \& } = \frac{1}{1-r} & \text{if } |r| < 1 \text{ (converges)} \\ \text{does \underline{not} exist} & \text{if } |r| \geq 1 \text{ (diverges)} \end{cases}$ *

- proof: we have $s_m = 1 + r + r^2 + \ldots + r^m = \frac{1 - r^{m+1}}{1-r}$

(why? $(1-r)(1 + r + r^2 + \ldots + r^m) = 1 - r^{m+1}$)

then $\lim_{m \to \infty} \frac{1-r^{m+1}}{1-r} = \begin{cases} \frac{1}{1-r} & \text{if } |r| < 1 \\ \text{diverges} & \text{if } |r| > 1 \end{cases}$

criterion for convergence of $\sum_{n=1}^{\infty} a_n$ (series):

converges & $a_n \to 0$

* theorem 6: 1) if $\sum_{n=1}^{\infty} a_n$ converges then $a_n \to 0$

2) if a_n diverges or $\lim_{n \to \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges *

ex) $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots =$ will diverge but $a_n \to 0$

($a_n \to 0$ doesn't always mean $\sum a_n$ converges)

$s_1 = 1$

$s_2 = 1.5$

$s_3 = 1.8$

$\vdots$